

ADOMIAN DECOMPOSITION METHOD FOR CONCENTRATION PROBLEM ARISING IN LONGITUDINAL DISPERSION OF MISCIBLE FLUID FLOW THROUGH POROUS MEDIA

Sandeep Shah

*Department of Applied Mathematics & Humanities
Sardar Vallabhbhai National Institute of Technology
Surat - 395007, Gujarat, INDIA
E-mail: sandeepshah7398@gmail.com*

Abstract—*In this paper a the problem is to find the concentration as a function of time 't' and position 'x' as the two miscible fluid flow through porous media on either sides of the mixed region. Miscible displacement in porous media plays a prominent role in many engineering and science fields such as oil recovery in petroleum engineering. The single fluid equation describes the motion of fluid. contamination of ground water by waste product disposed under ground movement of mineral in the soil and recovery of spent liquors in pulping process. Most of the works reveal common assumption of homogenous porous media with constant porosity, steady seepage flow velocity and constant dispersion coefficient. The Numerical Solution and graphical illustration of the dispersion problem is presented by means of Adomian Decomposition method for nonlinear partial differential equations.*

Keywords: *Adomian Decomposition method, Miscible fluid, Dispersion, Flows in porous media.*

1. INTRODUCTION

In a miscible displacement process a fluid is displaced in a porous medium by another fluid that is miscible with the first fluid. Miscible displacement in porous media plays a prominent role in many engineering and science fields such as oil recovery in petroleum engineering, contamination of ground water by waste product disposed underground movement of mineral in the soil and recovery of spent liquors in pulping process. Among Many flow problems in porous media, one involves fluid mixtures called miscible fluids. A miscible fluid is a single phase fluid consisting of several completely dissolved homogenous fluid species, a distinct fluid-fluid interface doesn't exist in a miscible fluid. The flow of miscible fluid is an important topic in petroleum industry; an enhanced recovery technique in oil reservoir involves injecting a fluid (solvent) that will dissolve the reservoir's oil. These problems of dispersion have been receiving considerable attention from chemical, environmental and

petroleum engineers, hydrologists, mathematicians and soil scientists.

Most of the works reveal common assumption of homogenous porous media with constant porosity, steady seepage flow velocity and constant dispersion coefficient. For such assumption Ebach and White [4] studied the longitudinal dispersion problem for an input concentration that varies periodically with time. Al-Niami and Rushton [1] studied the analysis of flow against dispersion in porous media. Hunt [6] applied the perturbation method to longitudinal and lateral dispersion in no uniform seepage flow through heterogeneous aquifers. Meher and Mehta [8, 9] studied the Dispersion of Miscible fluid in semi infinite porous media with unsteady velocity distribution from different point of view. The present chapter discusses the approximate solution of the nonlinear differential equation for longitudinal dispersion phenomena which takes places when miscible fluids mix in the direction of flow.

2. MATHEMATICAL FORMULATION AND SOLUTION OF THE PROBLEM

The problem is to find the concentration as a function of time 't' and position 'x' as the two miscible fluid flow through porous media on either sides of the mixed region . The single fluid equation describes the motion of fluid [14]. Here the mixing takes place longitudinally as well as transversely at $t = 0$ and a dot of fluid having $[C_0]$ concentration is injected over the phase. The dot moves in the direction of flow as well as perpendicular to the flow. Finally it takes the shape of the ellipse with a different concentration $[C_n]$.

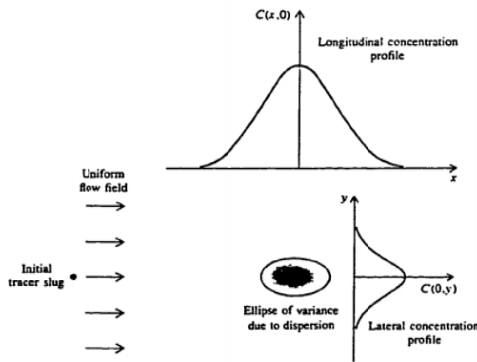


Figure 1.2.1 : Dispersion of an instantaneous point source in a uniform flow field

According to Darcy’s law the equation of continuity for the mixture in case of incompressible fluids is given by

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \bar{v}) = 0 \tag{1}$$

Where ‘ ρ ’ is the density for mixture and ‘ \bar{v} ’ is the pore seepage velocity.

The equation of diffusion for a fluid flow through a homogeneous porous medium without increasing or decreasing the dispersing material Polubarinova [10] is given by

$$\frac{\partial C}{\partial t} + \nabla \cdot (C \bar{v}) = \nabla \cdot \left[\rho \bar{D} \nabla \left(\frac{C}{\rho} \right) \right] \tag{2}$$

Where ‘ C ’ is the concentration of a fluid in a porous media. D is the Coefficient of dispersion with nine components D_{ij} . In a laminar flow for an Incompressible fluid through homogeneous porous medium, density ‘ ρ ’ is constant. Then equation (2) becomes,

$$\frac{\partial C}{\partial t} + \bar{v} \cdot \nabla C = \nabla \cdot (\bar{D} \nabla C) \tag{3}$$

Let us assume that the seepage velocity \bar{v} is along the x - axis, then $\bar{v} = u(x, t)$ and the non zero components will be $D_{11} \approx D_L = E$ (coefficient of longitudinal dispersion) and other Components will be zero Polubarinova[10]. Equation (3) becomes

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} = E \frac{\partial^2 C}{\partial x^2} \tag{4}$$

Where u is the component velocity along x -axis which is time dependent as well as concentration along x axis in $x \geq 0$ direction and $D_L > 0$ and it is the crosssectional flow velocity in porous media. $u = \frac{C(x,t)}{C_0}$, Where $x > 0$ and for $C_0 \equiv 1$, Mehta[9]. Equation (4) becomes

$$\frac{\partial C}{\partial t} + C \frac{\partial C}{\partial x} - E \frac{\partial^2 C}{\partial x^2} = 0 \tag{5}$$

This is the non linear Burger’s equation for longitudinal dispersion of miscible fluid flow through porous media. The theory that follows is confined to dispersion in unidirectional seepage flow through semi-infinite homogeneous porous media. The seepage flow velocity is assumed unsteady. The dispersion systems to be considered are subject to an input concentration of contaminants C_0 . The porous medium is considered as non-adsorbing. Consider the input concentration is C_0 . The governing partial differential equation (5) for longitudinal hydrodynamic dispersion with in a semi-infinite non-adsorbing porous medium in a unidirectional flow field in which E is the longitudinal dispersion coefficient, C is the average cross-sectional concentration, u is the unsteady seepage velocity, x is a coordinate parallel to flow and t is time.

The initial and boundary conditions are

$$C(x, 0) = C_0(x) = e^{-x}, x \geq 0 \tag{6}$$

Since Concentration is decreasing as x with distance x . Therefore for the sake of convenience $f(x)$ is considered as negative exponential function Mehta [9].

3. ADOMIAN DECOMPOSITION METHOD A THEORETIC APPROACH

We solve equation (4) for $L_t C(x, t)$ separately and we get

$$L_t C(x, t) = L_x C(x, t) - NC(x, t) \tag{7}$$

Let L_t^{-1} be the inverse operator of $L_t C(x, t)$ respectively ,given by the form

$$L_t^{-1} = \int (\cdot) dt \tag{8}$$

Then operating both sides of equation (7) and with the inverse operators (8) we obtain

$$C(x, t) = A + L_t^{-1} \left(E \frac{\partial^2 C}{\partial x^2} - C \frac{\partial C}{\partial x} \right) \tag{9}$$

Where A can be determined subjected to the corresponding initial and boundary condition (6) and we obtain:

$$C_0 = e^{-x}, \tag{10}$$

Now (10) , we get the following form

$$C(x, t) = \left[A + L_t^{-1} \left(E \frac{\partial^2 C}{\partial x^2} - C \frac{\partial C}{\partial x} \right) \right] \tag{11}$$

We write the parameterized form of (11) and the parameterized decomposition forms of C as

$$C(x, t) = \sum_{n=0}^{\infty} \lambda^n C_n(x, t) \tag{12}$$

Where A_n the Adomian special polynomial’s are to be determined. Here the parameter λ looks like a perturbation parameter. but actually is not a perturbation parameter; it is used only for grouping the terms. Now substitution of (12) gives

$$\sum_{n=0}^{\infty} \lambda^n C(x, t) = C_0(x, t) + \left[L_t^{-1} \left(E \frac{\partial^2 \sum_{n=0}^{\infty} \lambda^n C_n}{\partial x^2} - \sum_{n=0}^{\infty} \lambda^n C_n \right) \right] \quad (13)$$

If we compare like power terms of λ from both sides of equation (14) and taking under consideration that parameter λ is being proved that has the unique value $\lambda=0.5$ Cherruault [3] we get

$$\begin{aligned} C_0(x, t) &= e^{-x} \\ A_0 &= C \frac{\partial C}{\partial x} \\ C_1(x, t) &= L_t^{-1} \left[E \frac{\partial^2 C}{\partial x^2} - A_0 \right] \\ C_2(x, t) &= L_t^{-1} \left[E \frac{\partial^2 C}{\partial x^2} - A_1 \right] \\ C_n(x, t) &= L_t^{-1} \left[E \frac{\partial^2 C}{\partial x^2} - A_n \right] \end{aligned} \quad (14)$$

for $n = 0, 1, 2, 3, \dots, \dots, \dots, n$

Next we determine Adomian's special Polynomials A_n 's.

4. DETERMINATION OF ADOMIAN'S SPECIAL POLYNOMIALS

The A_n polynomials are determined in such a way that each A_n depend only on C_0, C_1, \dots, C_n for $n=0, 1, 2, 3, \dots, \dots, n$, i.e.

$$A_0 = A(C_0), A_1 = A_1(C_0, C_1), A_2 = A_2(C_0, C_1, C_2, C_3, C_4) \text{ etc.}$$

In order to do this we substitute (12) we have

From (12) we conclude that the Adomian polynomials have the following form:

$$\begin{aligned} A_0 &= C_0 \frac{\partial C_0}{\partial x} \\ A_1 &= C_0 \frac{\partial C_1}{\partial x} + C_1 \frac{\partial C_0}{\partial x} \\ A_2 &= C_0 \frac{\partial C_2}{\partial x} + C_1 \frac{\partial C_1}{\partial x} + C_2 \frac{\partial C_0}{\partial x} \end{aligned} \quad (15)$$

Hence, the polynomial A_0 has the following form:

$$A_0 = -e^{-2x}$$

$$C_1 = L_t^{-1} \left(E \frac{\partial C_0}{\partial x} - A_0 \right) \Rightarrow \int_0^t \left(E \cdot \frac{\partial^2}{\partial x^2} C_0 - A_0 \right) dt$$

$$C_1 = e^{-x} t (E + e^{-x})$$

$$C_2 = L_t^{-1} \left(E \frac{\partial C_1}{\partial x} - A_1 \right) \Rightarrow \int_0^t \left(E \cdot \frac{\partial^2}{\partial x^2} C_1 - A_1 \right) dt$$

$$C_2 = \frac{1}{2} e^{-x} (E^2 + 6E e^{-x} + 3(e^{-x})^2) t^2$$

$$C_3 = L_t^{-1} \left(E \frac{\partial C_2}{\partial x} - A_2 \right) \Rightarrow \int_0^t \left(E \cdot \frac{\partial^2}{\partial x^2} C_2 - A_2 \right) dt$$

$$C_3 = \frac{1}{6} e^{-x} (E^3 + 28E^2 e^{-x} + 51E(e^{-x})^2 + 16(e^{-x})^3) t^3$$

$$C_4 = L_t^{-1} \left(E \frac{\partial C_3}{\partial x} - A_3 \right) \Rightarrow \int_0^t \left(E \cdot \frac{\partial^2}{\partial x^2} C_3 - A_3 \right) dt$$

$$C_4 = \frac{1}{24} e^{-x} (E^4 + 120E^3 e^{-x} + 606E^2(e^{-x})^2 + 568E(e^{-x})^3 + 125(e^{-x})^4) t^4$$

If we suggest as a solution of C as an approximation of only two terms of the form

$$C = C_0 + C_1 + C_2 + C_3 + C_4$$

Simplify:

$$\begin{aligned} C = & \frac{1}{24} (E^4 t^4 + 4E^3 t^3 + 12E^2 t^2 + 24E t + 24) e^{-x} + \frac{1}{24} (120E^3 t^4 + 112E^2 t^3 + 72E t^2 \\ & + 24t) e^{-2x} + \frac{1}{24} (606E^2 t^4 + 204E t^3 + 36t^2) e^{-3x} + \frac{1}{24} (568E t^4 + 64t^3) e^{-4x} \\ & + \frac{125}{24} t^4 e^{-5x} \end{aligned} \quad (16)$$

Equation (16) represents the concentration at any point x for any time t during longitudinal dispersion in miscible displacement process.

5. RESULT AND DISCUSSION:

TABLE-1

x/t	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
0.1	0.9131823	0.9217519	0.9305462	0.939565	0.948809	0.958277	0.96797	0.977888	0.98803	0.9983981
0.2	0.8255582	0.8325523	0.8397131	0.847041	0.854535	0.862195	0.87002	0.878017	0.88618	0.8945049
0.3	0.7464052	0.7521158	0.75795	0.763908	0.769989	0.776194	0.78252	0.788975	0.79555	0.8022508
0.4	0.6748927	0.6795571	0.6843131	0.689161	0.6941	0.699132	0.70425	0.709469	0.71478	0.7201735
0.5	0.6102738	0.614085	0.6179642	0.621911	0.625927	0.63001	0.63416	0.638381	0.64267	0.6470241
0.6	0.5518763	0.5549914	0.558157	0.561373	0.56464	0.567957	0.57132	0.574743	0.57821	0.5817302
0.7	0.4990948	0.5016419	0.5042264	0.506848	0.509508	0.512205	0.51494	0.517711	0.52052	0.5233672
0.8	0.4513843	0.4534675	0.4555785	0.457717	0.459884	0.462078	0.4643	0.466551	0.46883	0.4711344
0.9	0.4082533	0.4099576	0.4116826	0.413428	0.415194	0.416981	0.41879	0.420617	0.42247	0.4243358
1	0.3692589	0.3706536	0.3720637	0.373489	0.37493	0.376386	0.37786	0.379344	0.38085	0.3823641

TABLE-2

x/t	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.01	0.9131823	0.9217519	0.9305462	0.939565	0.948809	0.958277	0.96797	0.977888	0.98803	0.9983981
0.02	0.8255582	0.8325523	0.8397131	0.847041	0.854535	0.862195	0.87002	0.878017	0.88618	0.8945049
0.03	0.7464052	0.7521158	0.75795	0.763908	0.769989	0.776194	0.78252	0.788975	0.79555	0.8022508
0.04	0.6748927	0.6795571	0.6843131	0.689161	0.6941	0.699132	0.70425	0.709469	0.71478	0.7201735
0.05	0.6102738	0.614085	0.6179642	0.621911	0.625927	0.63001	0.63416	0.638381	0.64267	0.6470241
0.06	0.5518763	0.5549914	0.558157	0.561373	0.56464	0.567957	0.57132	0.574743	0.57821	0.5817302
0.07	0.4990948	0.5016419	0.5042264	0.506848	0.509508	0.512205	0.51494	0.517711	0.52052	0.5233672
0.08	0.4513843	0.4534675	0.4555785	0.457717	0.459884	0.462078	0.4643	0.466551	0.46883	0.4711344
0.09	0.4082533	0.4099576	0.4116826	0.413428	0.415194	0.416981	0.41879	0.420617	0.42247	0.4243358
0.1	0.3692589	0.3706536	0.3720637	0.373489	0.37493	0.376386	0.37786	0.379344	0.38085	0.3823641

Table-1 and 2 represents the numerical values of concentration at any point x and for any time t>0.

Graph-1 0 < x < 1 for fixed t=0,0.01,.....

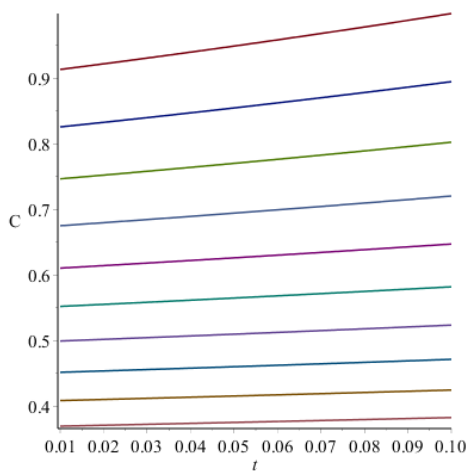


Figure 1:Concentration Vs. time

Graph-2 0 <t< 1 for fixed =0,0.1,...1

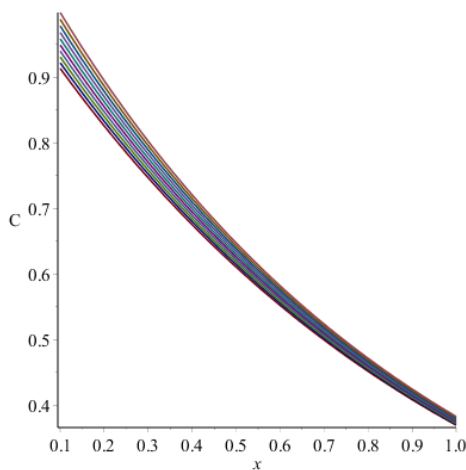


Figure 2: Concentration Vs. distance

Fig.1 and 2 discusses the variation of concentration with distance and time which shows that the value of concentration increases with time and decreases with distance in longitudinal dispersion during miscible displacement process.

6. CONCLUSION

Numerical and Graphical solutions have been developed for predicting the possible concentration of a given dissolved substance in an unsteady unidirectional seepage flows through semi-infinite, homogeneous, isotropic porous media subject to the source concentrations that vary negative exponentially with distance. Finally here we make a comparison of numerical solution with the approximate solution which shows Concentration decreases with distance and slightly increases with time. The solution express by equation (3.16) shows that approximate value of concentration (up to two terms) at any x for t>0. The analytical expressions obtained here are useful to the study of salinity intrusion in groundwater, helpful in making quantitative predictions on the possible contamination of groundwater supplies resulting from groundwater movement through buried wastes.

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